

Econ61001 Econometric Methods

Exercise Sheet 2

Len Gill

2013-2014

Contents

1	Introduction	2
2	Using Maple TA	2
3	Using the equation editor in Maple TA	3
4	Entering vectors and matrices	3
5	Question 1: Vectors	5
6	Question 2: 3 - d vectors	6
7	Question 3: Matrices	6
8	Question 4: Inner products	8
9	Question 5: Across and down	9
10	Question 6: Transposition	10
11	Question 7: Special Matrices	10
12	Question 8: Partitioned matrices	11
13	Question 9: Data matrices	11

1 Introduction

Blackboard is the University's virtual learning environment. All of the material for this module, including formative assessments, and summative assessments, will be available in Blackboard.

You will normally enter the Blackboard site for this course through MyManchester at <https://my.manchester.ac.uk/>.

All of the Blackboard material for the course is organised into Lecture topics, so that lecture notes, lecture slides, and exercise sheets can be found in the folder for each Lecture topic.

The questions on each exercise sheet may be answered in the traditional way, on paper, and handed in to be marked. Alternatively you can answer the majority (but not all) of the exercise sheet questions online, using Maple TA. All of the questions on this exercise sheet can be answered in Maple TA. The name of the matching Maple TA assignment is indicated in each question in this exercise sheet.

2 Using Maple TA

Maple T.A. is an easy-to-use web-based system for creating tests and assignments, and automatically assessing student responses and performance. In this module, most of the questions on the paper Exercise Sheets are also available as Maple TA assignments. The idea is that you can answer the questions at your leisure, and have them correctly graded by Maple TA. Once you have answered a question, the correct answers and/or sketch answers are immediately available. You can go back and attempt the same question as many times as you like.

Maple TA is located at

<http://place36.placementtester.com/manchester>

and this link is also given in each Lecture folder, for convenience. Login with your registration number (first 7 digits only): the password is also your registration number. Once you have logged in, there is generally a wait whilst the assignments are loaded. On the page that follows, you can click on MyProfile and then Password Update to change your password.

You should select the course

ECON61001 Econometric Methods 2013-14

by clicking on the entry for this course. This will bring up a page of assignments. You can click on the assignment you want to do - the notation follows that in the exercise sheets. The assignments are organised by question group, as in the Exercise Sheets, or by individual question - a component of the question group. Picking a question group means that you have to answer all the questions in that group before you can grade your answers. Picking an individual question enables you to grade your answers immediately.

When you click on an assignment, you are given choice between "Print assignment for off-line work" or "Work assignment on-line right now". If you choose to print, wait for the questions to be printed. When you have answered the questions, you can login and click on the assignment again, and choose the "Work ... online" option to enter your answers.

Usually, you are given information on the type of response (number, formula etc)

you are expected to give. If not, a textual response is required. In general, it is better to show arithmetic operators (+, −, *, /,) explicitly in your answers. Use of brackets to make clear meaning is also encouraged: what exactly is meant by “ $1/x - 1$ ” - is it $(1/x) - 1$ or $1/(x - 1)$? Information about the entry of vectors and matrices in your answers is given in the next section, although the instructions are often repeated in questions.

When you have finished one page of questions, click Next to go to the next part of the main question. You can also use the drop down menu of the Question item. When you have finished, click Grade and View Details to see the marked version of your answers. This screen also contains a Comments section, which gives sketch answers. You can also click on Quit and Save, or on Print.

You can save your work and return to it later if you clicked on Quit and Save when doing the assignment, but before clicking on Grade. To return, simply find the assignment in the Class Homepage list, and click on it.

To inspect completed and marked assignments, start from the Assignments page by clicking on Class Homepage if necessary. Click on Gradebook, and select View Past Results. Select the assignment you want to inspect, click on Search, find the assignment in the list at the bottom of the page, and click on Details.

Most of the Exercise Sheets have additional “randomised” questions associated with them in Maple TA. These are questions where Maple TA generates the numbers, which are different every time the question is attempted. These questions are intended for additional practice, should this be required, or for revision. These “randomised” questions are sometimes easier than, and sometimes harder than, the corresponding Exercise Sheet questions. If you find one of these randomised questions to be too hard, simply click on the “Refresh” button at the top of the page to get another question.

3 Using the equation editor in Maple TA

Usually it is quicker to enter your answers directly in Maple TA rather than using the Equation Editor. Using the Equation Editor is straightforward: see Figure 3.1. To select a symbol, click on the required panel and select the symbol required. Figure 3.2 shows the subscript and superscript panel, and Figure 3.3 shows the array selection panel. Usage is fairly self-evident - keep trying until you find the required symbol.

4 Entering vectors and matrices into Maple TA

Many questions on this exercise sheet require vectors, matrices or indexed expression as their answers. Some questions provide an array of the right size for the answer, so that none of the methods below are required.

There are a variety of ways of entering these expressions into Maple TA - choose the way you feel most comfortable in using.

1. Vector entry.

a) You can use Maple “text entry” for column vectors, e.g. for the 3×1 vector,

$$\begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix},$$

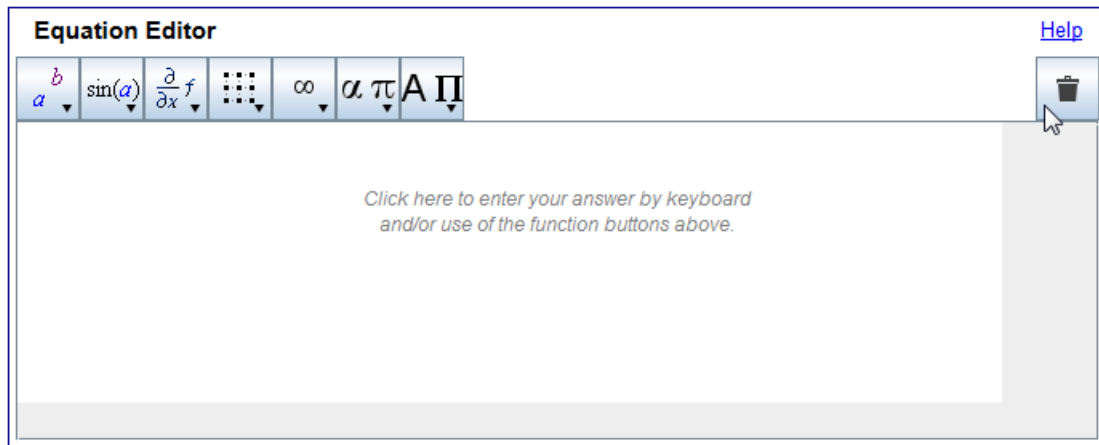


Figure 3.1: The Equation Editor in Maple TA

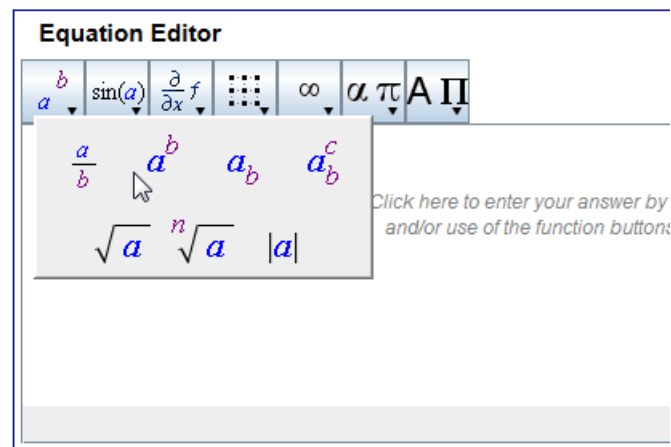


Figure 3.2: Subscripts and superscripts.

enter (without the quotes) `Vector([1,2,3])`.

- b) A quicker version is `< 1,2,3 >`.
- c) For a row vector, use `< 1|2|3 >`, as above, or `Vector[row]([1,2,3])`.
- d) Or, you can right click in the “Equation Editor”, select the array object, choose the correct matrix size and enter the elements of the vector.

2. Matrix entry.

- a) You can use Maple “text entry” for matrices, e.g. for a 2×3 matrix, enter (without the quotes) `Matrix([[1,2,3],[345]])` for the matrix

$$\begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{pmatrix}.$$

For more columns, include more elements in each `[.]` block. For more rows, add more `[.]` blocks, each separated by a comma. This is “row - by - row” construction.

- b) A quicker version is `< 1,2,3;4,5,6 >`.

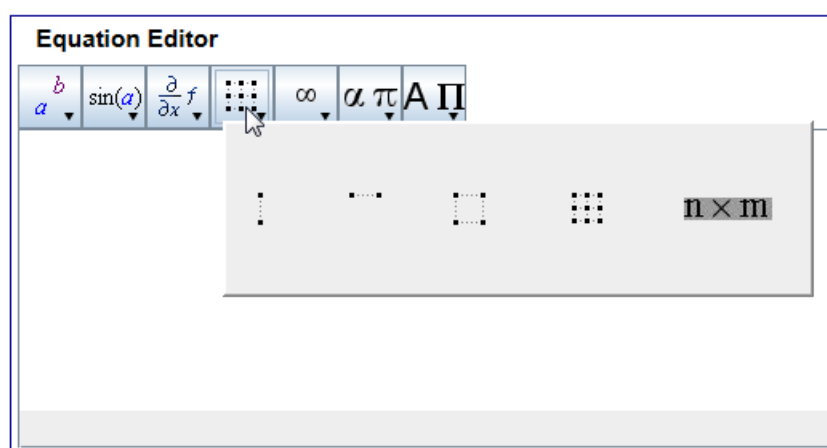


Figure 3.3: Arrays.

- c) Column by column construction is also possible using $\langle 1, 4 | 2, 5 | 3, 6 \rangle$ or $\langle \langle 1, 4 \rangle | \langle 2, 5 \rangle | \langle 3, 6 \rangle \rangle$.
 - d) A matrix with one row, say $\begin{bmatrix} 1 & 2 & 3 \end{bmatrix}$ can be entered as $\text{Matrix}([1, 2, 3])$, or as $\langle 1 | 2 | 3 \rangle$.
 - e) A matrix with one column, say $\begin{bmatrix} 1 \\ 4 \end{bmatrix}$, can be entered as $\text{Matrix}([[1], [4]])$ or as $\langle 1, 4 \rangle$.
 - f) Or, you can right click in the “Equation Editor”, select the array object, choose the correct matrix size and enter the elements of the product.
3. Entering indexed elements in vectors and matrices - the methods below can be used with any of the vector or matrix entry schemes.
- a) To insert x_1 into an answer, you can use $x[1]$, or you can use the Equation Editor. NB: select the right palette in the equation editor **before** entering any symbols!
 - b) To insert x_{11} , use $x[1,1]$, or use the Equation Editor, subject to the previous warning.

On the whole, the entry methods using $\langle \dots \rangle$ are the quickest to use.

Arithmetic operators are best shown explicitly in entering answers. These operators are the standard ones, $+$, $-$, $*$, $/$ and $^$.

5 Question 1: Vectors

1. (XS1 Q1 Part 1) Given the vectors

$$\mathbf{0} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}, \quad \mathbf{x} = \begin{bmatrix} -2 \\ 3 \end{bmatrix}, \quad \mathbf{y} = \begin{bmatrix} 3 \\ 5 \end{bmatrix}, \quad \mathbf{z} = \begin{bmatrix} 4 \\ 2 \end{bmatrix},$$

find

$$\mathbf{0} - \mathbf{x}, \mathbf{x} + \mathbf{y}, \mathbf{x} - \mathbf{y}, \mathbf{x} - 2\mathbf{z}, 3\mathbf{z} - 2\mathbf{y}, \mathbf{x} - 2\mathbf{y} + \mathbf{z}.$$

2. (XS1 Q1 Part 2) Can you find a linear combination $\alpha\mathbf{x} + \beta\mathbf{y}$ of $\mathbf{x} = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$ and $\mathbf{y} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$ to generate (in turn) the vectors

$$\mathbf{w}_1 = \begin{bmatrix} 4 \\ -2 \end{bmatrix}, \mathbf{w}_2 = \begin{bmatrix} -2 \\ 4 \end{bmatrix}, \mathbf{w}_3 = \begin{bmatrix} -2 \\ 2 \end{bmatrix}?$$

Additional “randomised” questions in Maple TA are in ExSheet2 Randomised Questions. The individual questions are XS2 Random Question 1 Part 1 and Part 2.

6 Question 2: 3 - d vectors

1. (XS1 Q2 Part 2) For the vectors

$$\mathbf{x} = \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix}, \quad \mathbf{y} = \begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix}$$

find $-2\mathbf{x}$, $3\mathbf{x} + 2\mathbf{y}$, $4\mathbf{x} - 2\mathbf{y}$.

2. (XS1 Q2 Part 2) For the vectors

$$\mathbf{x} = \begin{bmatrix} x_1 \\ \vdots \\ x_n \end{bmatrix}, \quad \mathbf{y} = \begin{bmatrix} y_1 \\ \vdots \\ y_n \end{bmatrix}$$

find $10\mathbf{x} - \alpha\mathbf{y}, \beta\mathbf{y}$.

3. (XS1 Q2 Part 3) Use the vector $\mathbf{x} = \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix}$ and the vector $\mathbf{z} = \begin{bmatrix} 4 \\ 2 \end{bmatrix}$ to find $2\mathbf{x} - \mathbf{z}$, $\mathbf{x} + 3\mathbf{z}$.

Additional “randomised” questions in Maple TA are in ExSheet2 Randomised Questions. The individual questions are XS2 Random Question 2 Part 1 and Part 2.

7 Question 3: Matrices

1. (XS1 Q3 Part 1) Arrange the vectors

$$\mathbf{0} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}, \quad \mathbf{x} = \begin{bmatrix} -2 \\ 3 \end{bmatrix}, \quad \mathbf{y} = \begin{bmatrix} 3 \\ 5 \end{bmatrix}, \quad \mathbf{z} = \begin{bmatrix} 4 \\ 2 \end{bmatrix}.$$

as the columns of a matrix A . What is the dimension of A ? Find a suitable vector α so that the product $A\alpha$ is equal to the following linear combinations in turn: $\mathbf{0} - \mathbf{x}$, $\mathbf{x} + \mathbf{y}$, $\mathbf{x} - \mathbf{y}$, $2\mathbf{z} - \mathbf{x}$, $\mathbf{y} - 2\mathbf{z}$, $\mathbf{x} + \mathbf{y} - \mathbf{z}$.

2. (XS1 Q3 Part 2) Arrange the vectors

$$\mathbf{x} = \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix}, \quad \mathbf{y} = \begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix}$$

as the columns of a matrix A . What is the dimension of A ? Find a suitable vector α so that the product $A\alpha$ is equal to the following linear combinations in turn: $-2\mathbf{x}$, $3\mathbf{x} + 2\mathbf{y}$, $4\mathbf{x} - 2\mathbf{y}$.

3. (XS1 Q3 Part 3) Arrange the vectors

$$\mathbf{x} = \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix}, \quad \mathbf{z} = \begin{bmatrix} 4 \\ 2 \end{bmatrix}$$

as the columns of a matrix A . What is the dimension of A ?

4. (XS1 Q3 Part 4) Arrange the vectors

$$\mathbf{x} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}, \quad \mathbf{y} = \begin{bmatrix} 1 \\ -1 \\ -1 \end{bmatrix}, \quad \mathbf{z} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$$

as the columns of a matrix A . What is the dimension of A ? If $\alpha = \begin{bmatrix} 2 \\ 1 \\ 3 \end{bmatrix}$ find $A\alpha$.

5. (XS1 Q3 Part 5) Arrange the vectors

$$\mathbf{x} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}, \quad \mathbf{y} = \begin{bmatrix} 1 \\ -1 \\ -1 \end{bmatrix}, \quad \mathbf{w} = \begin{bmatrix} -1 \\ -1 \\ 1 \end{bmatrix}$$

as the columns of a matrix A . What is the dimension of A ? If $\alpha = \begin{bmatrix} 0 \\ -1/2 \\ -1/2 \end{bmatrix}$ find $A\alpha$.

6. (XS1 Q3 Part 6) Arrange the vectors

$$\mathbf{x} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}, \quad \mathbf{y} = \begin{bmatrix} 1 \\ -1 \\ -1 \end{bmatrix}, \quad \mathbf{w} = \begin{bmatrix} -1 \\ -1 \\ 1 \end{bmatrix}, \quad \mathbf{v} = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$$

as the columns of a matrix A . What is the dimension of A ? If $\alpha = \begin{bmatrix} 0 \\ 1/2 \\ 1/2 \\ 1 \end{bmatrix}$ find $A\alpha$.

Additional “randomised” questions in Maple TA are in ExSheet2 Randomised Questions. The individual questions are XS2 Random Question 3 Part 1, Part 2, Part 4, Part 5 and Part 6.

8 Question 4: Inner products

1. (XS1 Q4 Part 1) Find the inner product of the vectors

$$\mathbf{x} = \begin{bmatrix} -2 \\ 3 \end{bmatrix}, \quad \mathbf{y} = \begin{bmatrix} 3 \\ 5 \end{bmatrix}.$$

What is $\mathbf{x}^T \mathbf{x}$ and $\mathbf{y}^T \mathbf{y}$? Is it true that $(\mathbf{x}^T \mathbf{y})^2 \leq \mathbf{x}^T \mathbf{x} \times \mathbf{y}^T \mathbf{y}$? This is called the Cauchy-Schwartz Inequality; equality holds only when $\mathbf{x} = \lambda \mathbf{y}$, for some λ .

2. (XS1 Q4 Part 2) Find a vector \mathbf{z} with the property that $\mathbf{x}^T \mathbf{z} = 0$ for $\mathbf{x} = \begin{bmatrix} -2 \\ 3 \end{bmatrix}$. Draw a diagram showing \mathbf{x} and \mathbf{z} : how would you describe the relationship between \mathbf{x} and \mathbf{z} ?

3. (XS1 Q4 Part 3) For

$$\mathbf{x} = \begin{bmatrix} 1/3 \\ 1/3 \\ 1/3 \end{bmatrix}, \quad \mathbf{y} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}, \quad \mathbf{z} = \begin{bmatrix} 6 \\ 2 \\ -1 \end{bmatrix}$$

find $\mathbf{x}^T \mathbf{y}$, $\mathbf{x}^T \mathbf{z}$, $\mathbf{y}^T \mathbf{z}$. How would you describe the value of these inner products from a statistical perspective?

4. (XS1 Q4 Part 4) If $\mathbf{x} = \begin{bmatrix} x_1 \\ \vdots \\ x_{10} \end{bmatrix}$, what is $\mathbf{x}^T \mathbf{x}$?

5. (XS1 Q4 Part 5) If $\mathbf{z} = \begin{bmatrix} 3 \\ 7 \\ 1 \\ 9 \end{bmatrix}$ and $\mathbf{1}_4$ is a 4×1 vector with every element equal to 1, what is the quantity $c = \frac{1}{4} \mathbf{1}_4^T \mathbf{z}$? Find the elements of the vector $\mathbf{z} - c \mathbf{1}_4$. From a statistical perspective, what are the elements of this vector? Find the value of the inner product $\mathbf{1}_4^T (\mathbf{z} - c \mathbf{1}_4)$. What statistical information does this illustrate?

6. (XS1 Q4 Part 6) Using $\mathbf{z} = \begin{bmatrix} 3 \\ 7 \\ 1 \\ 9 \end{bmatrix}$, $\mathbf{1}_4$ and c from part (5), what is the inner product of $(\mathbf{z} - c \mathbf{1}_4)$ with itself? If this distance is divided by 3 or 4, what statistical quantity is the result?

Additional “randomised” questions in Maple TA are in ExSheet2 Randomised Questions. The individual questions are XS2 Random Question 4 Part 1, Part 2, Part 3, and Part 4.

9 Question 5: Across and down

1. (XS1 Q5 Part 1) Suppose that B is an $r \times s$ matrix with typical element $\|b_{ij}\|$, and \mathbf{z} is an $s \times 1$ vector, with typical element z_j . What is the second element of the product $B\mathbf{z}$? What is the k th element?
2. (XS1 Q5 Part 2) If $A = \begin{bmatrix} 2 & 1 & 3 \\ 1 & 0 & 4 \end{bmatrix}$, and $\mathbf{b}_1 = \begin{bmatrix} 4 \\ 1 \\ 2 \end{bmatrix}$, $\mathbf{b}_2 = \begin{bmatrix} 1 \\ 2 \\ -1 \end{bmatrix}$, find $A\mathbf{b}_1$, $A\mathbf{b}_2$.
3. (XS1 Q5 Part 3) If $A = \begin{bmatrix} 2 & 1 & 3 \\ 1 & 0 & 4 \end{bmatrix}$, and $\mathbf{b}_1 = \begin{bmatrix} 4 \\ 1 \\ 2 \end{bmatrix}$, $\mathbf{b}_2 = \begin{bmatrix} 1 \\ 2 \\ -1 \end{bmatrix}$, make \mathbf{b}_1 and \mathbf{b}_2 the columns of a matrix B and find the matrix which is equal to AB .
4. (XS1 Q5 Part 4) If $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 4 \\ 6 & 7 \end{bmatrix}$, find AB and BA . What property do the matrices A and B have?
5. (XS1 Q5 Part 5) If $A_4 = \begin{bmatrix} -3 & 9 & 4 \\ 0 & 5 & 2 \end{bmatrix}$ and $A_{10} = \begin{bmatrix} -3 & 2 \\ 4 & 9 \\ 1 & -2 \end{bmatrix}$, find A_4A_{10} and $A_{10}A_4$. Why are A_4A_{10} and $A_{10}A_4$ not equal?
6. (XS1 Q5 Part 6) If $A_4 = \begin{bmatrix} -3 & 9 & 4 \\ 0 & 5 & 2 \end{bmatrix}$ and $A_{11} = \begin{bmatrix} 2 & 4 & 4 \\ 3 & 1 & -1 \\ 2 & 0 & 4 \end{bmatrix}$, find A_4A_{11} and $A_{11}A_4$.
7. (XS1 Q5 Part 7) If $A_{11} = \begin{bmatrix} 2 & 4 & 4 \\ 3 & 1 & -1 \\ 2 & 0 & 4 \end{bmatrix}$ and $A_{12} = \begin{bmatrix} 3 & 0 & -1 \\ -2 & -1 & -1 \\ 1 & 2 & 3 \end{bmatrix}$, find $A_{11}A_{12}$ and $A_{12}A_{11}$.
8. (XS1 Q5 Part 8) If $A_1 = \begin{bmatrix} 4 & 0 & -3 \\ 1 & -1 & 4 \\ 2 & 2 & -11 \end{bmatrix}$ and $A_8 = \begin{bmatrix} 4 & 2 & 6 \\ 1 & 3 & 4 \\ 5 & 0 & 5 \end{bmatrix}$, find $2A_8 - 3A_1$.

Additional “randomised” questions in Maple TA are in ExSheet2 Randomised Questions. The individual questions are XS2 Random Question 5 Part 1, Part 2, Part 4, and Part 8.

10 Question 6: Transposition

1. (XS1 Q6 Part 1) Find the transpose of the following matrices:

$$A_1 = \begin{bmatrix} -2 & 3 \\ 3 & 5 \end{bmatrix}; \quad A_2 = \begin{bmatrix} 1 & 0 & 9 \\ 6 & -2 & 15 \end{bmatrix}; \quad A_3 = \begin{bmatrix} 1 & 1 \\ 2 & -1 \\ 0 & 0 \end{bmatrix};$$

$$A_4 = \begin{bmatrix} 1 & 1 & -1 \\ 1 & -1 & -1 \\ 1 & -1 & 1 \end{bmatrix}; \quad A_5 = \begin{bmatrix} 1 & 1 & -1 \\ 1 & -1 & -1 \\ -1 & -1 & 1 \end{bmatrix}; \quad A_6 = \begin{bmatrix} 4 & 2 \\ 3 & 5 \\ -2 & 3 \end{bmatrix}.$$

2. (XS1 Q6 Part 1) What is the dimension of A^T for each matrix in part (1)?
3. Check that $(A^T)^T = A$ for each of the matrices in part (1).
4. (XS1 Q6 Part 1) What properties do the matrices A_1 and A_5 have that are not shared by the other matrices in part (1)?

5. (XS1 Q6 Part 5) If $A_4 = \begin{bmatrix} -3 & 9 & 4 \\ 0 & 5 & 2 \end{bmatrix}$ and $A_{10} = \begin{bmatrix} -3 & 2 \\ 4 & 9 \\ 1 & -2 \end{bmatrix}$, find A_4^T and A_{10}^T .

Confirm that $(A_4 A_{10})^T = A_{10}^T A_4^T$.

Additional “randomised” questions in Maple TA are in ExSheet2 Randomised Questions. The individual questions are XS2 Random Question 6 Part 1, Part 4, and Part 5.

11 Question 7: Special Matrices

1. (XS1 Q7 Part 1) If $A_4 = \begin{bmatrix} -3 & 9 & 4 \\ 0 & 5 & 2 \end{bmatrix}$, confirm that $I_2 A_4 = A_4 I_3$.
2. (XS1 Q7 Part 2) If $D = \begin{bmatrix} 3 & 0 \\ 0 & 5 \end{bmatrix}$, $A_4 = \begin{bmatrix} -3 & 9 & 4 \\ 0 & 5 & 2 \end{bmatrix}$ and $A_{10} = \begin{bmatrix} -3 & 2 \\ 4 & 9 \\ 1 & -2 \end{bmatrix}$, find DA_4 and $A_{10}D$. What pattern can you detect in the results?
3. (XS1 Q7 Part 3) If $A_4 = \begin{bmatrix} -3 & 9 & 4 \\ 0 & 5 & 2 \end{bmatrix}$, find $A_4 A_4^T$ and $A_4^T A_4$. Are these two matrices equal? What property do these two matrices possess?
4. (XS1 Q7 Part 4) If $\mathbf{x} = \begin{bmatrix} -2 \\ 3 \end{bmatrix}$ and $\mathbf{y} = \begin{bmatrix} 1 \\ -1 \\ -1 \end{bmatrix}$, find $\mathbf{x}\mathbf{x}^T$, $\mathbf{y}\mathbf{y}^T$. Are these symmetric? Do they equal $\mathbf{x}^T \mathbf{x}$, $\mathbf{y}^T \mathbf{y}$? Find \mathbf{xy}^T : is \mathbf{xy}^T equal to \mathbf{yx}^T or $\mathbf{y}^T \mathbf{x}$?
5. (XS1 Q7 Part 5) If $L = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 3 & 2 & 1 \end{bmatrix}$, show that L^T is an upper triangular matrix.

Additional “randomised” questions in Maple TA are in ExSheet2 Randomised Questions. The individual questions are XS2 Random Question 7 Part 3 and 4.

12 Question 8: Partitioned matrices

1. (XS1 Q8 Part 1) Write $A_2 = \begin{bmatrix} 1 & 2 & 1 & -3 \\ 6 & 0 & 10 & 9 \\ 2 & 0 & 3 & 4 \end{bmatrix}$ in the form $A_2 = [B_1 \ B_2]$, where both B_1 and B_2 have two columns. What are B_1 and B_2 ? Write $A_9 = \begin{bmatrix} 2 & 0 \\ -1 & 3 \\ 4 & -2 \\ 7 & 7 \end{bmatrix}$ as $A_9 = \begin{bmatrix} C_1 \\ C_2 \end{bmatrix}$, where both C_1 and C_2 have 2 rows. Express the product A_2A_9 in terms of B_1, B_2, C_1 and C_2 . Do the same for the product A_9A_2 , carefully stating the dimensions of any submatrices.

2. (XS1 Q8 Part 1) If $A_2 = \begin{bmatrix} 1 & 2 & 1 & -3 \\ 6 & 0 & 10 & 9 \\ 2 & 0 & 3 & 4 \end{bmatrix} = [B_1 \ B_2]$, where both B_1 and B_2 have two columns, and $\mathbf{z} = \begin{bmatrix} 5 \\ 1 \\ -3 \\ 2 \end{bmatrix} = \begin{bmatrix} \mathbf{z}_1 \\ \mathbf{z}_2 \end{bmatrix}$ where \mathbf{z}_i is 2×1 , given an expression for $A_2\mathbf{z}$ in terms of $B_1, B_2, \mathbf{z}_1, \mathbf{z}_2$.

3. (XS1 Q8 Part 3) If $A = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix}$ and $\mathbf{z} = \begin{bmatrix} \mathbf{z}_1 \\ \mathbf{z}_2 \end{bmatrix}$, where A_{11} is 2×2 , A_{12} is 2×1 , A_{21} is 1×2 , A_{22} is 1×1 and \mathbf{z}_1 is 1×1 , \mathbf{z}_2 is 2×1 , is the matrix A well defined? Does the product $A\mathbf{z} = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix} \begin{bmatrix} \mathbf{z}_1 \\ \mathbf{z}_2 \end{bmatrix} = \begin{bmatrix} A_{11}\mathbf{z}_1 + A_{12}\mathbf{z}_2 \\ A_{21}\mathbf{z}_1 + A_{22}\mathbf{z}_2 \end{bmatrix}$ exist in this form?

Additional “randomised” questions in Maple TA are in ExSheet2 Randomised Questions. The individual questions are XS2 Random Question 8 Part 1.

13 Question 9: Data matrices

A small data set on variables y and x is

$$D = \begin{bmatrix} 14 & 2 \\ 17 & 4 \\ 8 & 3 \\ 16 & 5 \\ 3 & 2 \end{bmatrix}.$$

1. (XS1 Q9 Part 1) Define a vector of observations on y and a matrix X so that the two variable regression model

$$y_i = \alpha + \beta x_i + u_i$$

can be represented, using the data in D , as

$$\mathbf{y} = X\delta + \mathbf{u}.$$

2. (XS1 Q9 Part 1) Compute for your choice of X , $X^T X$ and $X^T \mathbf{y}$. Check your answers in Matlab.

Additional “randomised” questions in Maple TA are in ExSheet2 Randomised Questions. The individual questions are XS2 Random Question 9 Part 1.